The stochastic extended path approach

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Motivations

- Nonlinearities can play an important role in macroeconomics: Irreversible investment, ZLB, Borrowing constraint, ...
- Standard local approximation techniques fail to produce reliable results in the presence of kinks.
- Deterministic, perfect forresight models can be solved with much greater accuracy than stochastic ones.
- The extended path approach aims to leverage the accuracy of deterministic methods in capturing (deterministic) nonlinearities.
- But it neglects the implications of future uncertainty. Is this a concern? Can we improve the EP approach?

Model to be solved

$$\mathbb{E}_t\left[f\left(y_{t-1}, y_t, y_{t+1}, \varepsilon_t\right)\right] = 0$$

- y an $n \times 1$ vector of endogenous variables
- $f : \mathbb{R}^{3n+q} \rightarrow \mathbb{R}^n$
- $\varepsilon_t \sim \mathcal{N}\left(0, \Sigma\right) \perp y_{\underline{t-1}}$
- $\exists\, y^\star$ such that $f\left(y^\star, y^\star, y^\star, 0\right) = 0$

Perfect foresight version

$$\begin{cases} f\left(y_{t-1}, y_t, y_{t+1}, \varepsilon_t\right) = 0 \\ f\left(y_{t+h-1}, y_{t+h}, y_{t+h+1}, 0\right) = 0 \quad h = 1, \dots, H-2 \\ f\left(y_{t+H-2}, y_{t+H-1}, y^\star, 0\right) = 0 \end{cases}$$

- For a long enough simulation, one can consider that for all practical purpose the system is back to equilibrium in period H.

⇒ Two value boundary problem with initial conditions for some variables (states) and terminal conditions for others (jumpings).

- In practice, one can use a Newton method to solve the equations of the model stacked over all periods of the simulation.

Perfect foresight model solver

- The unknowns:

$$Y_t = (y'_t, y'_{t+1}, \dots, y'_{t+H-1})' \quad \text{a } nH \times 1 \text{ vector}$$

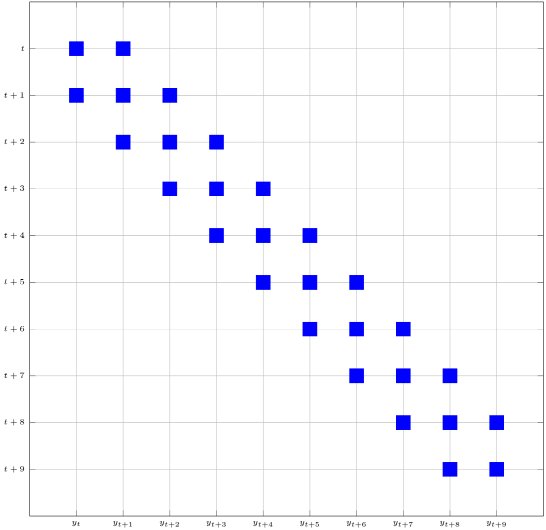
- We can rewrite the system of equations as $F(Y) = 0$, and solve it recursively:

$$Y_t^{(i+1)} = Y_t^{(i)} - J_F\left(Y_t^{(i)}\right)^{-1} F\left(Y_t^{(i)}\right)$$

- The jacobian J_F is potentially very large but **sparse**.

Stochastic perfect foresight model

Stacked jacobian, order=1, three nodes



Extended path

- Unexpected shocks in each period
- Loop over perfect foresight models

Algorithm 1 Extended path algorithm

1. $H \leftarrow$ Set the horizon of the perfect foresight (PF) model.
2. $y^\star \leftarrow$ Compute steady state of the model
3. $y_0 \leftarrow$ Choose an initial condition for the state variables
4. **for** $t = 1$ to T **do**
5. $\quad \varepsilon_t \leftarrow$ Draw random shocks for the current period
6. $\quad y_t \leftarrow$ Solve a PF with $y_{t+H} = y^\star$
7. **end for**

Approximate expectation

- Use gaussian quadrature

$$\mathbb{E}\left[\varphi(X)\right] = \int \varphi(x) f(x) \mathrm{d}x \approx \sum_{i=1}^{m} \omega_i \varphi(x_i)$$

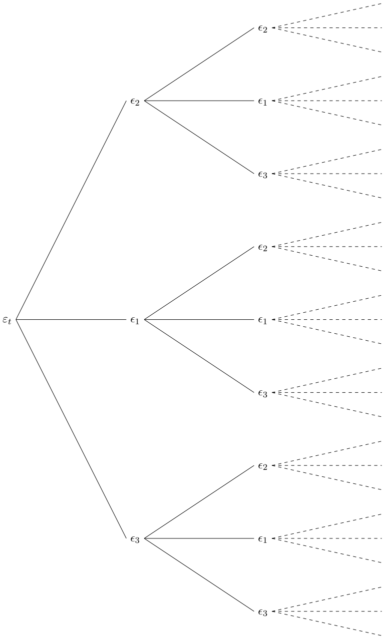
  where $(\omega_i, x_i)_{i=1}^{m}$ are the gaussian quadrature weight and nodes.

- If more than one source of future uncertainty, use tensor product (default in Dynare).

$\Rightarrow$ First curse of dimensionality.

- Use unscented transform (Julier et at., 2000): number of nodes grows linearly w.r.t the number of shocks.

Tree of forward histories (second curse)



Stochastic perfect foresight models

order 1

$$\sum_{i=1}^{m} \omega_i f\left(y_{t-1}, y_t, y_{t+1}^i, \varepsilon_t\right) = 0$$

$$\text{i=1,...,m} \begin{bmatrix} f\left(y_t, y_{t+1}^i, y_{t+2}^i, \epsilon_i\right) = 0 \\ f\left(y_{t+1}^i, y_{t+2}^i, y_{t+3}^i, 0\right) = 0 \\ \vdots \\ f\left(y_{t+H-2}^i, y_{t+H-1}^i, y^\star, 0\right) = 0 \end{bmatrix}$$

Stochastic perfect foresight models

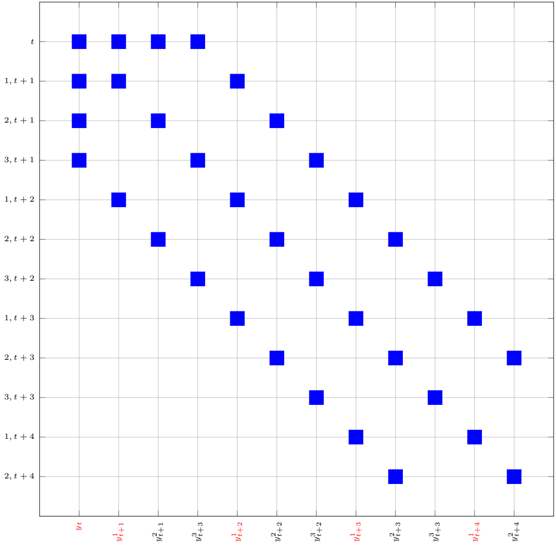
order 1

$$\begin{array}{cccc}
y_t & & & \\
y^1_{t+1} & y^2_{t+1} & \cdots & y^m_{t+1} \\
y^1_{t+2} & y^2_{t+2} & \cdots & y^m_{t+2} \\
\vdots & \vdots & & \vdots \\
y^\star & y^\star & \cdots & y^\star
\end{array}$$

$\rightarrow \quad 1 + m(H-1)$ unknown vectors $(n \times 1)$.

Stochastic perfect foresight model

Stacked jacobian, order=1, three nodes



Stochastic perfect foresight models

order 2

$$\sum_{i=1}^{m} \omega_i f\left(y_{t-1}, y_t, y_{t+1}^i, \varepsilon_t\right) = 0$$

$$\text{i=1,...,m}\left[\begin{array}{l} \sum_{j=1}^{m} \omega_j f\left(y_t, y_{t+1}^j, y_{t+2}^{j,i}, \epsilon_i\right) = 0 \\ \text{j=1,...,m}\left[\begin{array}{l} f\left(y_{t+1}^i, y_{t+2}^{j,i}, y_{t+3}^{j,i}, \epsilon_j\right) = 0 \\ f\left(y_{t+2}^{j,i}, y_{t+3}^{j,i}, y_{t+4}^{j,i}, 0\right) = 0 \\ \vdots \\ f\left(y_{t+H-2}^{j,i}, y_{t+H-1}^{j,i}, y^\star, 0\right) = 0 \end{array}\right. \end{array}\right.$$

Stochastic perfect foresight models

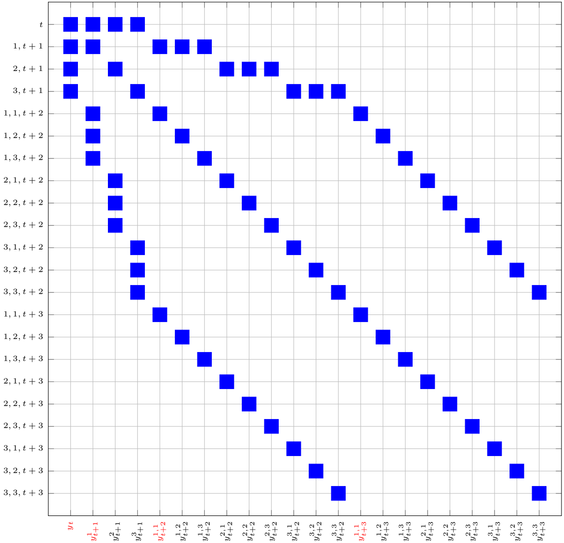
order 2

$$
\begin{array}{ccccccccccccc}
y_t & & & & & & & & & & & & \\
y^1_{t+1} & & & & y^2_{t+1} & & & & \cdots & y^m_{t+1} & & & \\
y^{1,1}_{t+2} & y^{2,1}_{t+2} & \cdots & y^{m,1}_{t+2} & y^{1,2}_{t+2} & y^{2,2}_{t+2} & \cdots & y^{m,2}_{t+2} & \cdots & y^{1,m}_{t+2} & y^{2,m}_{t+2} & \cdots & y^{m,m}_{t+2} \\
y^{1,1}_{t+3} & y^{2,1}_{t+3} & \cdots & y^{m,1}_{t+3} & y^{1,2}_{t+3} & y^{2,2}_{t+3} & \cdots & y^{m,2}_{t+3} & \cdots & y^{1,m}_{t+3} & y^{2,m}_{t+3} & \cdots & y^{m,m}_{t+3} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \cdots & \vdots & \vdots & & \vdots \\
y^\star & y^\star & \cdots & y^\star & y^\star & y^\star & \cdots & y^\star & \cdots & y^\star & y^\star & \cdots & y^\star \\
\\
\begin{pmatrix}\varepsilon_t\\ \epsilon_1\\ \epsilon_1\end{pmatrix} & \begin{pmatrix}\varepsilon_t\\ \epsilon_1\\ \epsilon_2\end{pmatrix} & \cdots & \begin{pmatrix}\varepsilon_t\\ \epsilon_1\\ \epsilon_m\end{pmatrix} & \begin{pmatrix}\varepsilon_t\\ \epsilon_2\\ \epsilon_1\end{pmatrix} & \begin{pmatrix}\varepsilon_t\\ \epsilon_2\\ \epsilon_2\end{pmatrix} & \cdots & \begin{pmatrix}\varepsilon_t\\ \epsilon_2\\ \epsilon_m\end{pmatrix} & \cdots & \begin{pmatrix}\varepsilon_t\\ \epsilon_m\\ \epsilon_1\end{pmatrix} & \begin{pmatrix}\varepsilon_t\\ \epsilon_m\\ \epsilon_2\end{pmatrix} & \cdots & \begin{pmatrix}\varepsilon_t\\ \epsilon_m\\ \epsilon_m\end{pmatrix} \\
\\
\omega_1^2 & \omega_1\omega_2 & \cdots & \omega_1\omega_m & \omega_2\omega_1 & \omega_2^2 & \cdots & \omega_2\omega_m & \cdots & \omega_m\omega_1 & \omega_n\omega_2 & \cdots & \omega_m^2
\end{array}
$$

$\rightarrow \quad 1 + m + m^2(H-2)$ unknown vectors $(n \times 1)$.

Stochastic perfect foresight model

Stacked jacobian, order=2, three nodes



Stochastic perfect foresight models

order p

$$
\begin{aligned}
&\sum_{i_1=1}^{m} \omega_{i_1} f\left(y_{t-1}, y_t, y_{t+1}^{i_1}, \varepsilon_t\right) = 0 \\
&\sum_{i_2=1}^{m} \omega_{i_2} f\left(y_t, y_{t+1}^{i_1}, y_{t+1}^{i_2,i_1}, \epsilon_{i_1}\right) = 0 \; \forall \, i_1 \in \{1,\dots,m\} \\
&\sum_{i_3=1}^{m} \omega_{i_3} f\left(y_{t+1}^{i_1}, y_{t+2}^{i_2,i_1}, y_{t+3}^{i_3,i_2,i_1}, \epsilon_{i_2}\right) = 0 \; \forall \, (i_1, i_2) \in \{1,\dots,m\}^2 \\
&\sum_{i_4=1}^{m} \omega_{i_4} f\left(y_{t+2}^{i_2,i_1}, y_{t+3}^{i_3,i_2,i_1}, y_{t+3}^{i_4,\dots,i_1}, \epsilon_{i_3}\right) = 0 \; \forall \, (i_1, i_2, i_3) \in \{1,\dots,m\}^3 \\
&\qquad\quad \vdots \\
&\sum_{i_p=1}^{m} \omega_{i_p} f\left(y_{t+p-2}^{i_{p-2},\dots,i_1}, y_{t+p-1}^{i_{p-1},\dots,i_1}, y_{t+p}^{i_p,\dots,i_1}, \epsilon_{i_{p-1}}\right) = 0 \; \forall \, (i_1,\dots,i_{p-1}) \in \{1,\dots,m\}^{p-1} \\
&\qquad\quad f\left(y_{t+p-1}^{i_{p-1},\dots,i_1}, y_{t+p}^{i_p,\dots,i_1}, y_{t+p+1}^{i_p,\dots,i_1}, \epsilon_{i_p}\right) = 0 \; \forall \, (i_1,\dots,i_p) \in \{1,\dots,m\}^p \\
&\qquad\quad f\left(y_{t+p}^{i_p,\dots,i_1}, y_{t+p+1}^{i_p,\dots,i_1}, y_{t+p+2}^{i_p,\dots,i_1}, 0\right) = 0 \; \forall \, (i_1,\dots,i_p) \in \{1,\dots,m\}^p \\
&\qquad\quad \vdots \\
&\qquad\quad f\left(y_{t+H-2}^{i_p,\dots,i_1}, y_{t+H-1}^{i_p,\dots,i_1}, y^{\star}, 0\right) = 0 \; \forall \, (i_1,\dots,i_p) \in \{1,\dots,m\}^p
\end{aligned}
$$

Stochastic perfect foresight models

order p

- Perfect m-ary tree of height p.
- The root, at height 0, is $\sum_{i_1=1}^{m} \omega_{i_1} f\left(y_{t-1}, y_t, y_{t+1}^{i_1}, \varepsilon_t\right)$.
- All the nodes from height 1 to $p-1$ are approximate integrals.
- The m^p terminal nodes at height p are deterministic problems.
- The m^p leafs are deterministic paths to the steady state.
- The size of the tree increases exponentially with the approximation order (p) and polynomially with the number of quadrature points (m).

Stochastic perfect foresight models

order p

- The number of unknown vectors is:

$$
\begin{aligned}
\mathcal{C}(m,p,H) &= 1 + m + m^2 + \ldots + m^{p-1} + m^p(H-p)\\
&= \frac{m^p-1}{m-1} + m^p(H-p)
\end{aligned}
$$

$\Rightarrow$ The jacobian square matrix has $n\mathcal{C}(m,p,H)$ columns.

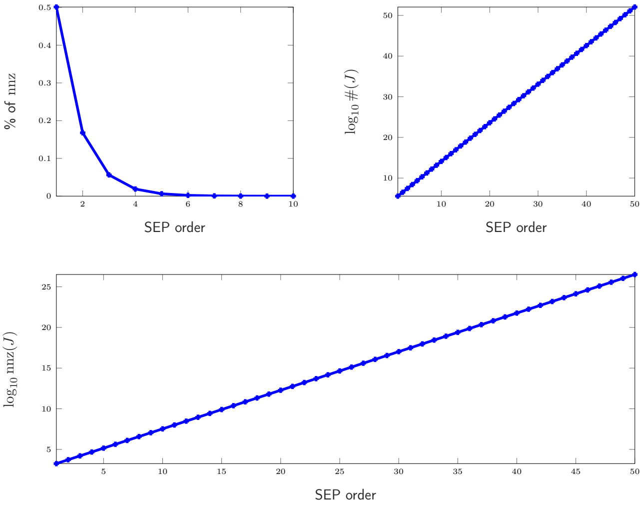
- Number of non zero $n \times n$ blocks is:

$$
\mathrm{nnz}(m,p,H) = 1 + m + (2+m)\frac{m^p - m}{m-1} + 3m^p(H-p) - m^p
$$

- The size of the Jacobian increases exponentially with the order of the SEP, while the sparsity improves (the proportion of non-zero blocks tends to zero as either m or p approaches infinity).

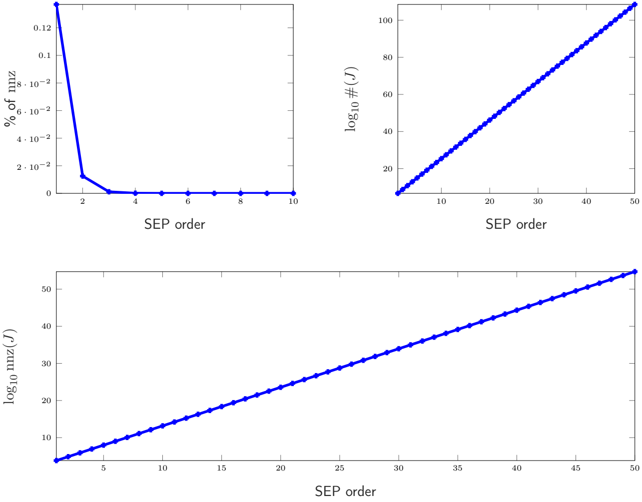
Stochastic perfect foresight models

Stacked jacobian sparsity (3 quadrature nodes)



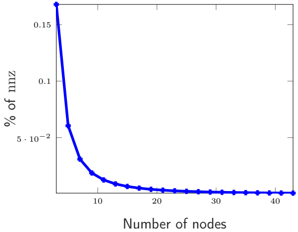
Stochastic perfect foresight models

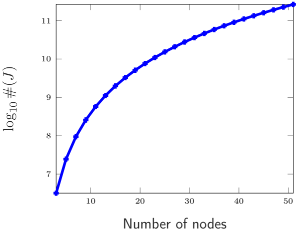
Stacked jacobian sparsity (11 quadrature nodes)

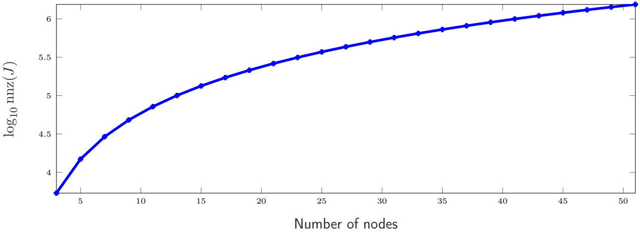


Stochastic perfect foresight models

Stacked jacobian sparsity (order 2)

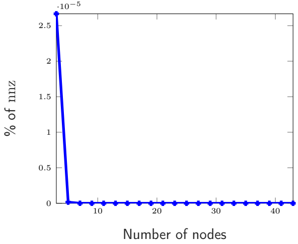


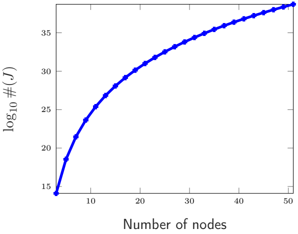


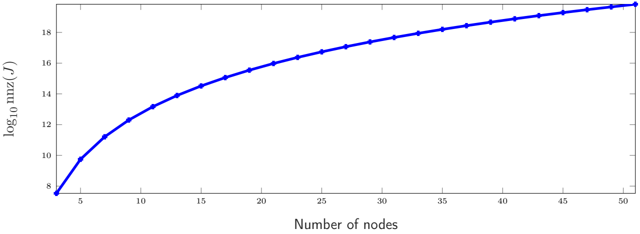


Stochastic perfect foresight models

Stacked jacobian sparsity (order 10)







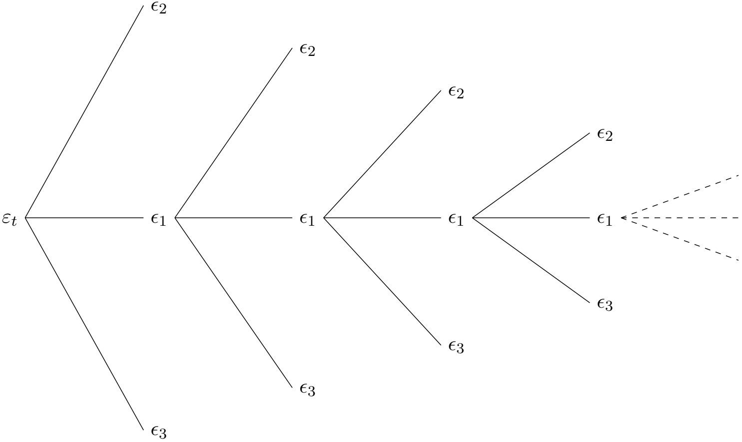
Stochastic perfect foresight model

Sparse tree

- Employing the complete m-ary tree presented above is infeasible for large values of p or m.
- Trimming the tree by eliminating branches with low probabilities (as determined by the products of quadrature weights) offers limited benefits, as the pruned tree would still expand exponentially with respect to p.
- The trunk of the m-ary tree is defined by traversing the central nodes from one period to the next.
- All branches that do not directly stem from the trunk are removed.
- Fishbone tree $\Leftrightarrow$ Monomial rule, where innovations in different periods are treated as distinct shocks.

Tree of forward histories

Sparse tree



Stochastic perfect foresight model

Sparse tree

- The leaf on each terminal node is a deterministic path to the steady state.
- The number of terminal nodes, mp, expands linearly with the SEP approximation order.
- The number of unknown $n \times 1$ vectors to be solved for is:

$$\begin{aligned}\bar{\mathcal{C}}(m,p,H) &= H + \sum_{i=1}^{p}(m-1)(H-i)\\ &= \left(1+(m-1)p\right)H - \frac{p(p+1)}{2}\end{aligned}$$

Stochastic perfect foresight model

Sparse tree

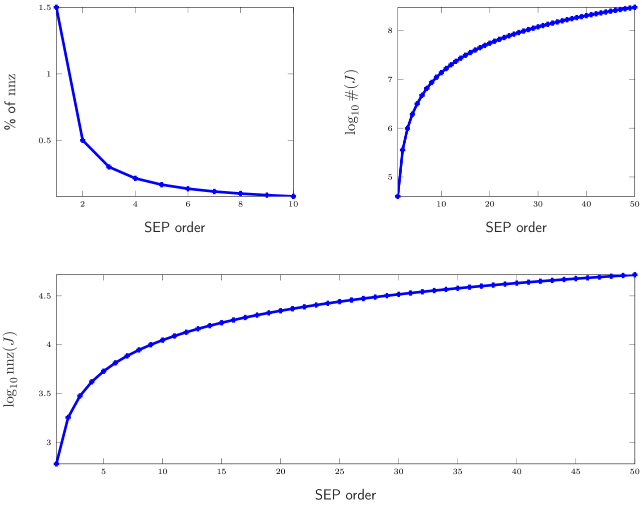
- The number of non zero $n \times n$ blocks in the stacked Jacobian is:

$$\overline{\mathrm{nnz}}(m,p,H) = \underbrace{\overbrace{3H-2}^{\mathrm{EP}} + \overbrace{(m-1)p}^{\textbf{Approximate integrals}}}_{\text{Along the trunk}} + (m-1)\sum_{i=1}^{p-1}(3(H-1-i)+2)$$

- The Jacobian is smaller in size, with its growth occurring linearly in relation to p or m; however, it is denser.

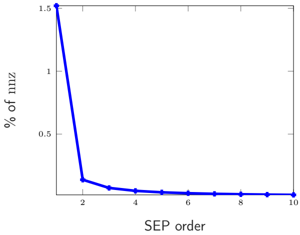
Stochastic perfect foresight models

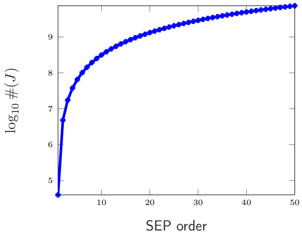
Stacked jacobian sparsity with sparse tree (3 quadrature nodes)

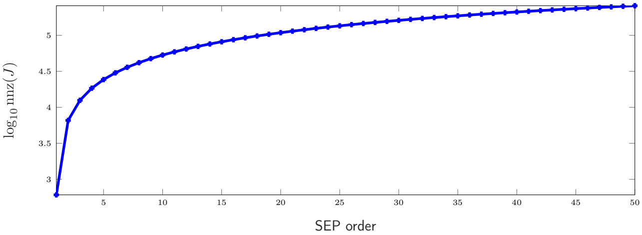


Stochastic perfect foresight models

Stacked jacobian sparsity with sparse tree (11 quadrature nodes)







Stochastic perfect foresight model

Sparse tree, equations for SEP(p)

- Let $y_{t,s}^{i}$ represent the vector of endogenous variables at time $s > t$ along a branch that diverges from the trunk at time t due to the anticipated shock (integration node) ϵ_i.

- The sequence y_t (the root), $y_{t,t+1}^{1}$, $y_{t+1,t+2}^{1}$, $\dots$, $y_{t+p-1,t+p}^{1}$ represents the path of the endogenous variables along the trunk.

- Approximate integrals are located along the trunk.

- In period t we have:

$$\sum_{i=1}^{m} \omega_i f\left(y_{t-1}, y_t, y_{t,t+1}^{i}, \varepsilon_t\right) = 0$$

Stochastic perfect foresight model

Sparse tree, equations for SEP(p)

- In period $t+1$ we have:

$$\sum_{i=1}^{m} \omega_i f\left(y_t, y^1_{t,t+1}, y^i_{t+1,t+2}, \epsilon_1\right) = 0$$
$$f\left(y_t, y^i_{t,t+1}, y^i_{t,t+2}, 0\right) = 0 \quad \forall\, i \in \{2, \dots, m\}$$

- In period $t+2$ we have:

$$\sum_{i=1}^{m} \omega_i f\left(y^1_{t,t+1}, y^1_{t+1,t+1}, y^i_{t+2,t+3}, \epsilon_1\right) = 0$$
$$f\left(y^i_{t,t+1}, y^i_{t,t+2}, y^i_{t,t+3}, 0\right) = 0 \quad \forall\, i \in \{2, \dots, m\}$$
$$f\left(y^1_{t,t+1}, y^i_{t+1,t+2}, y^i_{t+1,t+3}, 0\right) = 0 \quad \forall\, i \in \{2, \dots, m\}$$

Stochastic perfect foresight model

Sparse tree, equations for SEP(p)

- In period $t+h$, with $h<p$, we have:

$$
\begin{aligned}
&\sum_{i=1}^{m}\omega_i f\left(y^1_{t+h-2,t+h-1}, y^1_{t+h-1,t+h}, y^i_{t+h,t+h+1}, \epsilon_1\right) = 0\\
&f\left(y^i_{t,t+h-1}, y^i_{t,t+h}, y^i_{t,t+h+1}, 0\right) = 0 \quad \forall\, i\in\{2,\dots,m\}\\
&f\left(y^i_{t+1,t+h-1}, y^i_{t+1,t+h}, y^i_{t+1,t+h+1}, 0\right) = 0 \quad \forall\, i\in\{2,\dots,m\}\\
&\vdots\\
&f\left(y^1_{t+h-2,t+h-1}, y^i_{t+h-1,t+h}, y^i_{t+h-1,t+h+1}, 0\right) = 0 \quad \forall\, i\in\{2,\dots,m\}
\end{aligned}
$$

Stochastic perfect foresight model

Sparse tree, equations for SEP(p)

- In period $t+p$ we have:

$$
\begin{aligned}
&f\left(y^1_{t+p-2,t+p-1}, y^i_{t+p-1,t+p}, y^i_{t+p-1,t+p+1}, \epsilon_i\right) = 0 \quad \forall\, i \in \{1,\dots,m\}\\
&f\left(y^i_{t,t+p-1}, y^i_{t,t+p}, y^i_{t,t+p+1}, 0\right) = 0 \quad \forall\, i \in \{2,\dots,m\}\\
&f\left(y^i_{t+1,t+p-1}, y^i_{t+1,t+p}, y^i_{t+1,t+p+1}, 0\right) = 0 \quad \forall\, i \in \{2,\dots,m\}\\
&\vdots\\
&f\left(y^i_{t+p-2,t+p-1}, y^i_{t+p-1,t+p}, y^i_{t+p-1,t+p+1}, 0\right) = 0 \quad \forall\, i \in \{2,\dots,m\}
\end{aligned}
$$

Stochastic perfect foresight model

Sparse tree, equations for SEP(p)

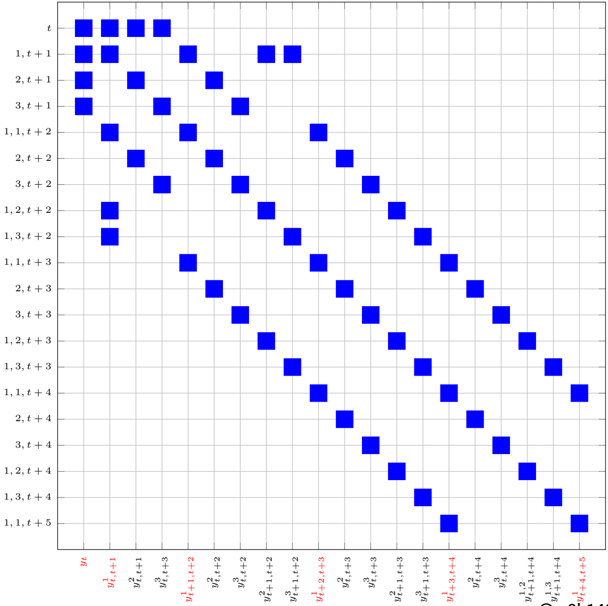
- In period $t+h$, with $h > p$, we have:

$$
\begin{aligned}
&f\left(y^i_{t,t+h-1}, y^i_{t,t+h}, y^i_{t,t+h+1}, 0\right) = 0 \quad \forall\, i \in \{2,\dots,m\}\\
&f\left(y^i_{t+1,t+h-1}, y^i_{t+1,t+h}, y^i_{t+1,t+h+1}, 0\right) = 0 \quad \forall\, i \in \{2,\dots,m\}\\
&\vdots\\
&f\left(y^i_{t+h-2,t+h-1}, y^i_{t+h-1,t+h}, y^i_{t+h-1,t+h+1}, 0\right) = 0 \quad \forall\, i \in \{2,\dots,m\}\\
&f\left(y^1_{t+h-2,t+h-1}, y^1_{t+h-1,t+h}, y^1_{t+h-1,t+h+1}, 0\right) = 0
\end{aligned}
$$

With $y^i_{s,t+H} = y^\star$ for all i and $t < s < t+H$.

Stochastic perfect foresight model

Stacked jacobian, sparse tree, order=2, three nodes



Burnside (1998) asset pricing model

- The price/dividend ratio and the growth rate of dividends:

$$y_t = \beta \mathbb{E}_t \left[e^{\theta x_{t+1}} \left(1 + y_{t+1}\right) \right]$$
$$x_t = (1-\rho)\bar{x} + \rho x_{t-1} + \epsilon_t$$

- The exact solution is:

$$y_t = \sum_{i=1}^{\infty} \beta^i e^{a_i + b_i \hat{x}_t}$$

where

$$a_i = \theta \bar{x} i + \frac{\theta^2 \sigma^2}{2(1-\rho)^2} \left(i - 2\rho \frac{1-\rho^i}{1-\rho} + \rho^2 \frac{1-\rho^{2i}}{1-\rho^2} \right)$$

and

$$b_i = \frac{\theta \rho \left(1 - \rho^i\right)}{1-\rho}$$

Numerical simulation

Calibration

$$\bar{x} = 0.0179$$
$$\rho = -0.139$$
$$\theta = -1.5$$
$$\beta = 0.95$$
$$\sigma = 0.0348$$

- The deterministic steady state is equal to 12.3035.
- The risky steady state, defined as the fix point in absence of shock this period:

$$\widetilde{y} = \sum_{i=1}^{\infty} \beta^i e^{\theta \bar{x} i + \frac{\theta^2 \sigma^2}{2(1-\rho)^2}\left(i - 2\rho \frac{1-\rho^i}{1-\rho} + \rho^2 \frac{1-\rho^{2i}}{1-\rho^2}\right)} \approx 12.4812$$

Comparing SEP, perturbation and closed-form solution

Simulate long time series ($T = 8000$) and compare with true solution. We use a quadrature with 3 nodes for SEP.

	P(1)	P(2)	SEP(0)	SEP(2)
$100 \times \mathrm{mean}(\lvert \hat{y}_t - y_t \rvert / y_t)$	1.4261	0.0193	1.4241	1.2534
$100 \times \max(\lvert \hat{y}_t - y_t \rvert / y_t)$	1.4707	0.0527	1.4250	1.2539

One can show, using the closed for solution and considering an infinite number of weights and nodes in the quadrature, that we would have to consider an approximation order greater than 60, to be able to recover the theoretical mean of the price-dividend ratio.

Hybrid approach

- Consider a Taylor expansion of the original problem only along the scale σ of the shocks
- Use the second order correction for the constant.

$\Rightarrow$ For K-order SEP, replace y_{t+K+1} in the equations in period K by $y_{t+K+1} + \frac{1}{2} g_{\sigma\sigma}$.

	P(2)	SEP(2)	SEP(2+)	SEP(10+)
$100 \times \mathrm{mean}(\lvert\hat{y}_t - y_t\rvert / y_t)$	0.0193	1.2534	0.0165	0.0153
$100 \times \max(\lvert\hat{y}_t - y_t\rvert / y_t)$	0.0527	1.2539	0.0179	0.0170

Irreversible investment

Consider the following RBC model with irreversible investment:

$$\max_{\{c_{t+j}, l_{t+j}, k_{t+j+1}\}_{j=0}^{\infty}} \quad \mathcal{W}_t = \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, l_{t+j})$$

$$\underline{s.t.}$$

$$\begin{aligned}
y_t &= c_t + i_t \\
y_t &= A_t f(k_t, l_t) \\
k_{t+1} &= i_t + (1-\delta)k_t \\
A_t &= A^\star e^{a_t} \\
a_t &= \rho a_{t-1} + \varepsilon_t \\
i_t &\geq 0
\end{aligned}$$

Further specifications

The utility function is

$$u(c_t, l_t) = \frac{\left(c_t^\theta (1 - l_t)^{1-\theta}\right)^\tau}{1 - \tau}$$

and the production function,

$$f(k_t, l_t) = \left(\alpha k_t^\psi + (1 - \alpha) l_t^\psi\right)^{\frac{1}{\psi}}$$

First order conditions

$$
\begin{aligned}
u_c(c_t, l_t) - \mu_t &= \beta \mathbb{E}_t \Big[u_c(c_{t+1}, l_{t+1}) \Big(A_{t+1} f_k(k_{t+1}, l_{t+1}) + 1 - \delta \Big) - \mu_{t+1}(1-\delta) \Big] \\
\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} &= A_t f_l(k_t, l_t) \\
c_t + k_{t+1} &= A_t f(k_t, l_t) + (1-\delta) k_t \\
0 &= \mu_t \left(k_{t+1} - (1-\delta) k_t \right)
\end{aligned}
$$

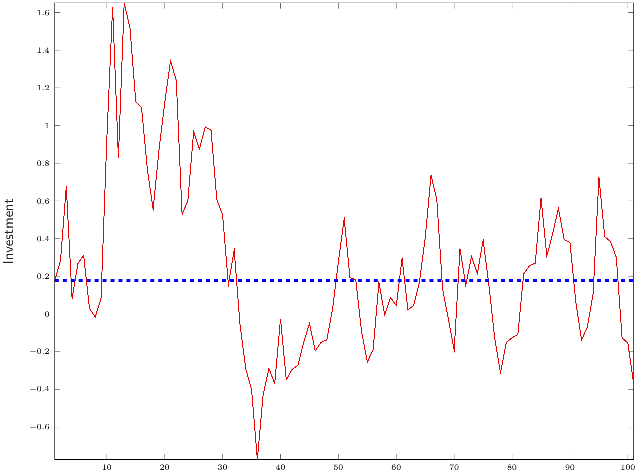
where μ_t is the Lagrange multiplier associated with the constraint on investment.

Calibration

$$\beta = 0.990$$
$$\theta = 0.357$$
$$\tau = 2.000$$
$$\alpha = 0.450$$
$$\psi = -0.500$$
$$\delta = 0.010$$
$$\rho = 0.800$$
$$A^\star = 1.000$$
$$\sigma = 0.100$$

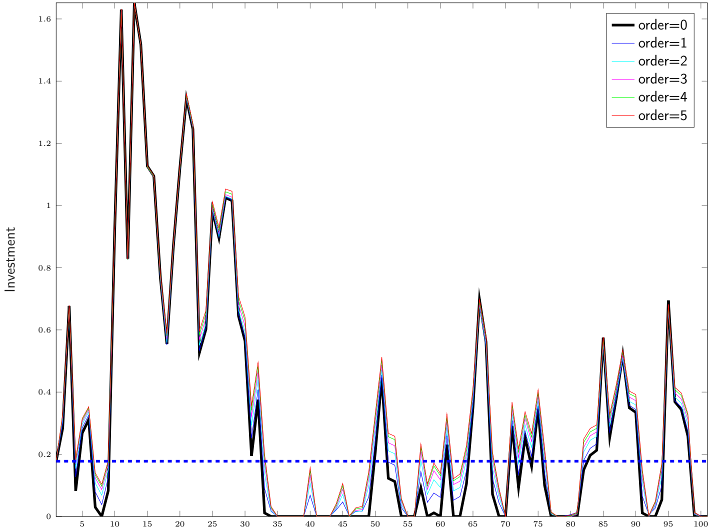
Simulation of the RBC model

SEP with orders=0,...,10



Simulation of the RBC model (irreversible investment)

SEP with orders=0,. . . ,5



Simulation of the RBC model (irreversible investment)

SEP with orders=0,. . . ,10